72. **Answer (C):** In order for $ad - bc$ to be odd, ad and bc must have opposite parity. Thus either ad is odd and bc is even, or vice versa. If ad is odd, then a and d must both be odd, which occurs with probability $\frac{5}{10} \cdot \frac{5}{10} = \frac{1}{4}$. In this case, bc must be even, and this occurs with probability $1 - \frac{1}{4} = \frac{3}{4}$. Thus the probability that ad is odd and bc is even is $\frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16}$. Similarly, the probability that ad is even and bc is odd is also $\frac{3}{16}$. Thus the probability that $ad - bc$ is odd is $\frac{3}{16} + \frac{3}{16} = \frac{3}{8}$.

73. **Answer (E):** Note that $\frac{7}{9} + \frac{2}{3} = (\sin^2 \alpha + \cos^2 \beta) + (\sin^2 \beta + \cos^2 \gamma) = 1 + \sin^2 \alpha + \cos^2 \gamma$, so $\sin^2 \alpha + \cos^2 \gamma = \frac{4}{9}$. Rewrite the last equation as $(1 - \cos^2 \alpha) + (1 - \sin^2 \gamma) = \frac{4}{9}$, or equivalently, $\sin^2 \gamma + \cos^2 \alpha = \frac{14}{9}$. Squaring each side of the given equation $\sin \gamma + \cos \alpha = \frac{2\sqrt{7}}{3}$ yields $\sin^2 \gamma + 2(\sin \gamma)(\cos \alpha) + \cos^2 \alpha = \frac{28}{9}$. Thus $(\sin \gamma)(\cos \alpha) = \frac{1}{2}\left(\frac{28}{9} - \frac{14}{9}\right) = \frac{7}{9}$.

76. *Answer: 32* The lengths of the legs are $\log 2^3 = 3 \log 2$ and $\log 2^4 = 4 \log 2$, so the length of the hypotenuse is $\sqrt{9(\log 2)^2 + 16(\log 2)^2} = 5 \log 2 = \log 2^5 = \log 32$, and $x = 32$.

77. **Answer (C):** Since x_1 and x_2 are solutions to a quadratic equation,

$$x_1 + x_2 = -4a$$
$$x_1 \cdot x_2 = -4.5a$$

Squaring both sides of the first equation and subtracting the second one multiplied by 2, we get

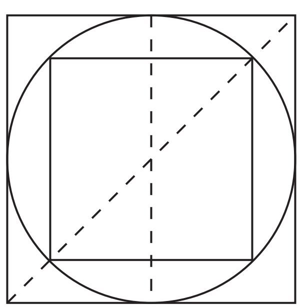
$$x_1^2 + x_2^2 = 16a^2 + 9a$$

Since we know that $x_1^2 + x_2^2 = 25$, we get this equation

$$16a^2 + 9a - 25 = 0$$

where a is unknown. The standard formula for quadratic equations leads to these solutions: 1 and -25/16.

78. **Answer (E):** Assume that the axes of C_1 and C_2 are vertical. Then, we have this figure in any plane containing the axes:



Let R be the radius of the sphere and H and h are the heights of C_1 and C_2, respectively. Then

$$H = 2R \qquad \text{and} \qquad h = \frac{2R}{\sqrt{2}}.$$

Therefore, $\frac{H}{h} = \sqrt{2}$.
Hence, the ratio of the volumes is

$$(\sqrt{2})^3 = 2\sqrt{2}.$$

79. *Answer: 2013* The first time, the population doubled in 6 years. According to the properties of geometric progressions, it will double again in another 6 years and reach 12 million in 2013.

80. *Answer: 7* If 2013 is the sum of the n consecutive positive integers starting with x, then

$$N = x + (x+1) + (x+2) + \cdots + (x+n-1) = nx + \frac{(n-1)n}{2},$$

so $n(n+2x-1) = 4026 = 2 \cdot 3 \cdot 11 \cdot 61$. The possible choices for n are therefore $2, 3, 6, 11, 22, 33$, and 61 (the next larger factor of 2013, namely 66, would necessarily result in the product $n(n+2x-1)$ being greater than 2013. The corresponding values of x, namely $((2013/n)-n+1)/2$, are $1006, 670, 333, 178, 8, 45$, and 3

81. *Answer: 5* If the base on the left side is 1, then the equation is satisfied. In that case, $1 = x^2 - 5x + 5$, so $0 = x^2 - 5x + 4 = (x-1)(x-4)$, and $x = 1$ or 4.

If the base on the left side is -1, then the equation is satisfied if and only if the exponent on the left side is an even integer. In that case it is necessary that $-1 = x^2 - 5x + 5$, so $0 = x^2 - 5x + 6 = (x-3)(x-2)$, and $x = 2$ or 3. However, the exponent on the left side is odd when $x = 3$, so the only solution is $x = 2$.

If the base on the left side is neither 1 nor -1, then the equation is satisfied if and only if the exponent on the left side is 0 and the base is not 0. In that case it is necessary that $0 = x^2 + 8x + 12 = (x+2)(x+6)$, and $x = -2$ or -6. Because the base on the left side is not 0 for either value of x, both are solutions.

Thus the equation has the five solutions -6, -2, 1, 2, and 4.

82. **Answer (C):** For any integer n, the expression $(n-1)(n)(n+1)(n+2)+1$ can be written as

$$\begin{aligned} n^4 + 2n^3 - n^2 - 2n + 1 &= n^4 + 2n^3 + n^2 - 2n^2 - 2n + 1 \\ &= (n^2+n)^2 - 2(n^2+n) + 1 \\ &= (n^2+n-1)^2. \end{aligned}$$

Therefore $\sqrt{2009 \cdot 2010 \cdot 2011 \cdot 2012 + 1} = 2010^2 + 2010 - 1$. The rightmost three digits are the same as those of $10^2 + 10 - 1 = 109$.

85. **Answer (B):** The surface area of the hemisphere is $(1/2)(4\pi(2)^2) = 8\pi$. The lateral surface area of the cone is $\pi \cdot 2 \cdot \sqrt{2^2 + 2^2} = 4\sqrt{2}\pi$. The total surface area is $\pi(8 + 4\sqrt{2})$.